

## NUMERICAL INVESTIGATION OF THE EFFECT OF THE IRREGULAR NATURE OF SEISMIC LOADING ON THE LIQUEFACTION RESISTANCE OF SATURATED SAND DEPOSITS

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**Abstract:** It is common practice in laboratory tests for the assessment of the liquefaction potential of sands to convert the irregular acceleration time-history to an equivalent number of uniform stress cycles. For this, a number of methodologies exist which are mainly based on Miner's (1945) accumulated damage concept. Miner's theory states that the damage that a material undergoes is not affected by the location of a cycle of stress within the loading history. However, soil is a nonlinear material with a stress path-dependent response. The composition of the loading history in terms of stress cycles and their magnitude should therefore have an influence on sands' liquefaction resistance. To shed more light on this, the recorded base ground motion from the Chi Chi, Taiwan, seismic event of the 20<sup>th</sup> September 1999 is used in non-linear elasto-plastic plane strain effective stress-based finite element analyses of a hypothetical homogeneous soil deposit consisting of Nevada Sand. The irregular surface acceleration time-history is then converted to an equivalent number of uniform as well as non-uniform amplitude cycles at different percentages of the maximum shear stress and single element undrained simple shear tests are numerically simulated. The soil response is investigated to assess the accuracy of the empirical procedures on the response of sands to liquefaction. The results suggest that the empirical methods may lead to non-conservative conclusions.

### Introduction

Extensive laboratory testing has been carried out over the years aiming to understand the deformation patterns and the pore water pressure development characteristics of sands and ultimately to better interpret earthquake-induced soil phenomena, such as the triggering of liquefaction and the development of settlements. Although earthquakes consist of irregular multi-directional components of ground motion, due to the difficulty and high cost of carrying out tests under irregular multi-directional shearing in the laboratory, element testing is normally carried out under uniform, uni-directional loading. The evaluated cyclic resistance ratio (CRR) of soils is therefore then corrected for load irregularity and multiplicity effects by applying relevant correction factors (Ishihara, 1996), according to Equation 1:

$$CRR_{ir, mult} = C_2 * C_5 * CRR \quad (1)$$

where  $C_2$  is the coefficient defined to account for the irregular nature of loading in one direction and  $C_5$  is the coefficient to correct for multi-directional loading (Ishihara, 1996). Ishihara and Nagase (1988) attempted to quantify the load irregularity and multiplicity effects on the liquefaction resistance of sands by performing a number of simple shear tests using six time histories of irregular loading applied in two perpendicular directions on the horizontal plane. The results of these tests for various sand densities showed that the irregular nature of loading increases the cyclic strength of loose sands by a factor of about 1.8, reducing to a value of approximately 1.2 at a relative density of 90%. An alternative approach to account

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for load irregularity, is to use the equivalent number of uniform stress cycles to estimate the seismic demand (CSR), which is then compared against the CRR evaluated from the uniform uni-directional laboratory element tests.

Numerous studies have focussed on the procedure of converting an irregular time-history to an equivalent number of constant amplitude cycles. Most of these are based on the widely used methodology by Seed et al. (1975), in which the peaks of the acceleration or shear stress time-history are counted and weighting factors are used to obtain the equivalent number of cycles of a chosen shear stress level. A schematic representation of the various steps to obtain the weighting factors for a reference stress level  $a$  is shown in Figure 1. According to this procedure, the effect of the application of one cycle at a stress level of  $b$  will be equivalent to the effect that would result if  $N_a/N_b$  cycles at a reference stress level  $a$  are applied. Cycles of stresses of an amplitude of less than 30% of the maximum shear stress,  $\tau_{max}$ , are neglected, as the corresponding weighting factors are insignificant. These factors are determined based on the curve of CRR against number of cycles to liquefaction ( $N_L$ ), obtained from the large scale undrained cyclic simple shear tests of DeAlba et al. (1975) on reconstituted Monterey No. 0 sand samples of a relative density of 65%. The curve is normalised by the maximum shear stress causing liquefaction in one cycle to obtain a representative shape, which is independent of a number of parameters, such as soil fabric, density and mean effective stress. A factor of safety of 1.5 has also been incorporated in the curve. The methodology is based on the assumption of proportionality between accelerations and stresses within the first 7m of a soil deposit (Seed and Idriss, 1971) and thus the equivalent number of stress cycles can be readily obtained from surface acceleration time-histories with no signs of liquefaction.

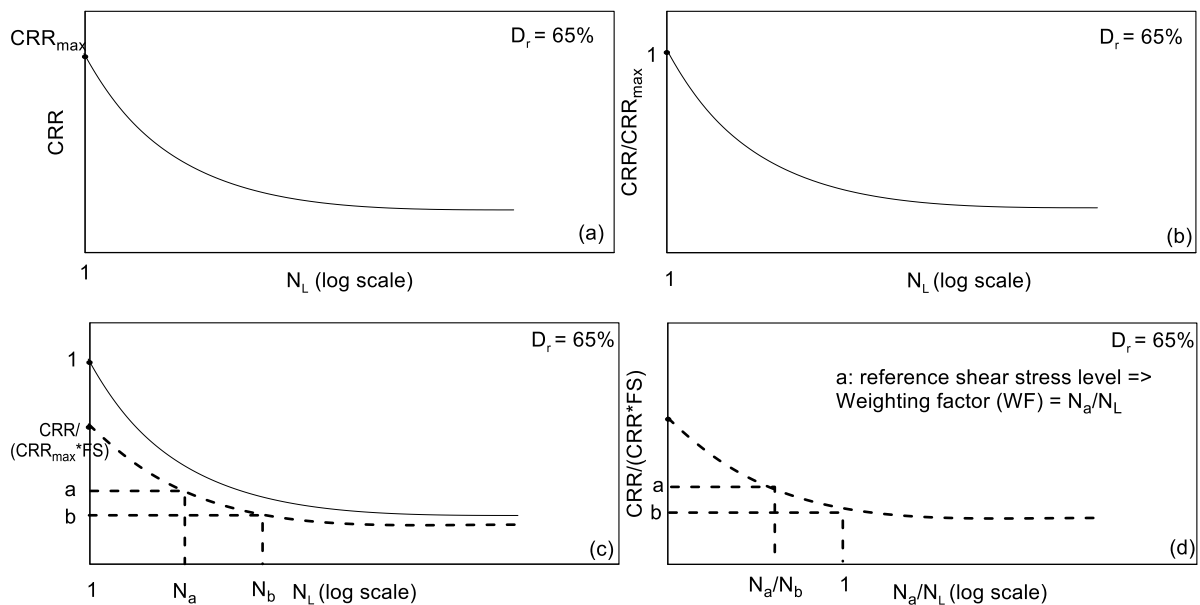


Figure 1. Seed et al. (1975) methodology for the conversion of an irregular stress time-history to an equivalent number of uniform stress amplitude cycles in liquefaction analyses

It has been custom to convert the irregular surface acceleration time-history to an equivalent number of uniform cycles of a shear stress amplitude corresponding to 65% of the maximum shear stress,  $\tau_{max}$ , registered at the point of interest. However, as stated by Seed et al. (1975), the selection of this percentage does not have a rigorous basis and, therefore, any other stress level could be used. This is based on the assumption that any point on the normalised CRR curve used to obtain the weighting factors is equivalent, i.e. liquefaction can be triggered by any of these points.

Clearly, the methodology has a number of inherent assumptions and is based on the original proposal by Miner (1945) for high cycle fatigue conditions of metals, being more applicable to large number of cycles of low amplitude during which the behaviour of the material is mainly elastic. For this reason, a number of researchers questioned over the years the applicability of this methodology for soils subjected to strong ground motion during which large irreversible strains develop (Shen et al., 1978; Azeteiro et al., 2012; Coelho et al., 2013). All of them conducted undrained cyclic triaxial tests and concluded that the location of a single peak shear stress within a loading pattern of otherwise uniform stress amplitude cycles affects significantly the liquefaction resistance of sands, contradicting Miner's (1945) accumulated damage concept; the earlier the maximum peak is applied during the loading time-history, the more the resistance of sands to liquefaction is reduced with an abrupt loss of effective stress. This implies that the evaluation of the liquefaction resistance of sands based on uniform stress amplitude testing could lead to non-conservative design. Azeteiro et al. (2012) and Coelho et al. (2013) also showed that the cyclic triaxial strength of sand is influenced substantially by the irregularity of the loading history even at loose states, contradicting the results of Shen et al. (1978).

In all cases, however, the peak stress cycle was large enough to cross the Phase Transformation Line - PTL (Ishihara, 1996), resulting in large plastic deformations and fabric changes. As such, the aim of the current study is to further investigate this phenomenon numerically by focussing on cycles of constant shear stress amplitude of different percentages of  $T_{max}$ , as well as by studying the influence of the location of a peak stress cycle within uniform amplitude cycles that does not necessarily cross the PTL. Site response analyses are first carried out to obtain the response of a sandy deposit to the applied earthquake loading. The resulting surface acceleration time-history is then converted into an equivalent number of uniform and non-uniform stress amplitude cycles, which are applied to numerical single element simulations of undrained simple shear tests (USS).

### Sand deposit and input ground motion

A hypothetical soil deposit consisting of Nevada Sand (NS) with a thickness of 15 m, a relative density of 43% and a permeability of  $6.5E-05$  m/s was considered in the performed finite element analyses. Nevada Sand is a clean uniform fine sand, which was used in the large collaborative VELACS centrifuge testing programme (Taborda, 2011). The sand deposit was assumed to be fully saturated with the water table specified at ground level, underlain by impermeable rigid bedrock. The deposit is shown in Figure 2.

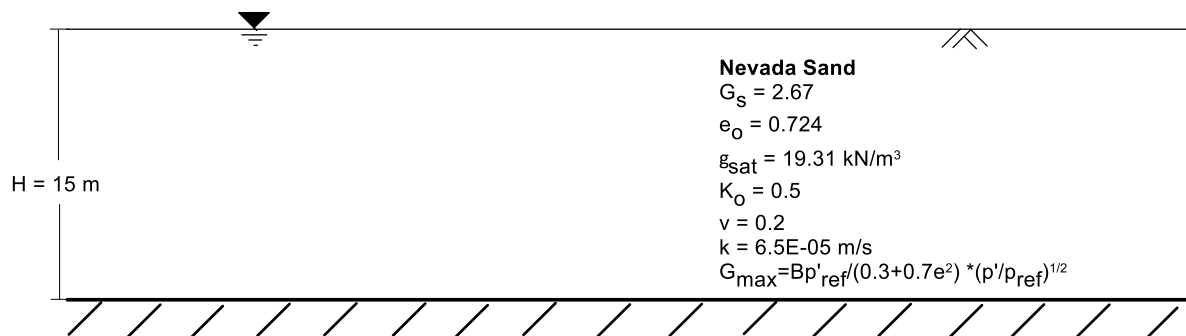


Figure 2. Sand deposit used in the FE analyses

The ground motion used in this study is the one recorded in the east-west (EW) direction at 52 m depth in a downhole array at the Hualien site during the 20<sup>th</sup> September 1999 Chi Chi, Taiwan, seismic event, with a Richter magnitude of  $M_L = 7.3$  and a peak ground acceleration (PGA) of  $0.4 \text{ m/s}^2$ . The acceleration time-history and the corresponding Fourier Spectrum are shown in Figure 3. A linear polynomial baseline correction was applied to the input motion using the computer software SeismoSignal v5.0.0 (Seismosoft, 2014).

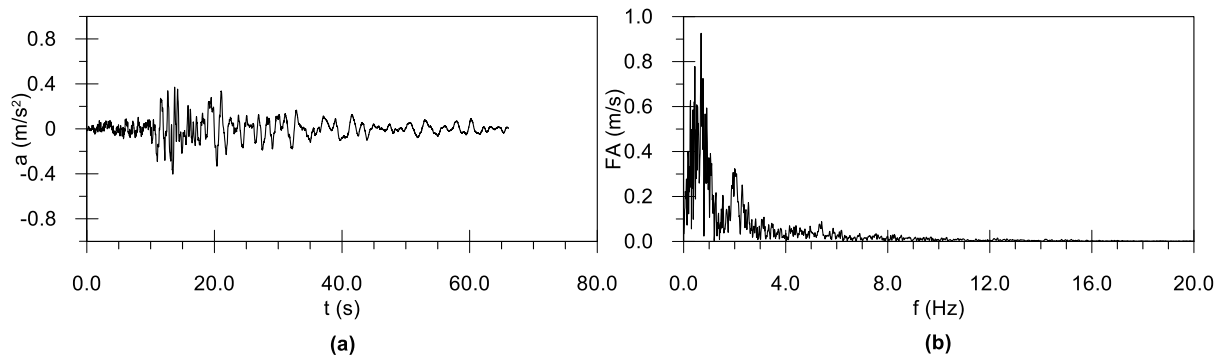


Figure 3. (a) Acceleration time-history of Chi-Chi, Taiwan, EW component recorded during the 20<sup>th</sup> September 1999 earthquake and (b) respective Fourier Spectrum

### Finite element analyses

Two site response analyses and eight undrained simple shear test (USS) simulations with different combinations of stress cycles were carried out in this study. The first site response analysis was carried out with full hydro-mechanical coupling, in order to predict the response of the 15 m deep sand deposit to liquefaction. Conversely, the second one was drained, aiming to obtain the surface acceleration time-history and convert it to an equivalent number of uniform and non-uniform stress cycles according to the Seed et al. (1975) methodology. The ‘peak-between-mean crossing count’ method was employed to count the peaks (Dowling, 1971). Undrained simple shear test numerical simulations were then carried out, as these are considered to be a more realistic representation of the field conditions when shear waves propagate upwards, compared to cyclic triaxial tests. The USS tests were carried out for the initial stress conditions of an element of soil at 7 m depth in the NS deposit. Two of these were carried out with uniform shear stress amplitude at 0.65 and 0.8 of  $\tau_{max}$  at 7 m depth, whereas in the remaining six tests a shear stress peak corresponding to  $\tau_{max}$  was employed in the loading, with its location varying between the beginning, the end and the middle of the total number of cycles. Azeteiro et al. (2012) analysed a number of acceleration time-histories of past earthquakes and by using two different criteria found that the maximum acceleration,  $a_{max}$ , is directly proportional to the average value,  $a_{ave}$ . However, depending on the criterion chosen, the coefficient of proportionality can be very different: one criterion yields a value of 0.65 (hence matching the one proposed by Seed et al. (1975)), whereas the other one a value of 0.37. Therefore, half of the tests with the peak shear stress cycle were simulated with the remaining cycles at an amplitude of  $0.65\tau_{max}$ , whereas in the other half the respective amplitude was taken as  $0.5\tau_{max}$  (i.e. at the average value of the ratio resulting from the two criteria).

Table 1 lists the characteristics of the simulated USS tests.

Table 1. Undrained simple shear test simulations

Test	Total No of shear stress cycles	Amplitude (% of $\tau_{max}$ )	Location of single peak stress cycle of amplitude $\tau_{max}$ within the loading sequence
0.65U	6	65	N/A
0.65PS	4	65	Start
0.65PE	4	65	End
0.65PM	4	65	Middle
0.5PS	15	50	Start
0.5PE	15	50	End
0.5PM	15	50	Middle
0.8U	3.5	80	N/A

### Numerical method and constitutive model

All finite element analyses are plane strain non-linear elasto-plastic effective stress-based and were carried out with the Imperial College Finite Element Program – ICFEP (Potts and Zdravković, 1999). The mesh generated for analysing the sand deposit consists of a column of 60x1 8-noded quadrilateral elements with dimensions of 0.25x0.25 m<sup>2</sup>. The choice of the height of the elements was based on the recommendations by Bathe (1996) for 8-noded solid elements, in order to ensure that the mesh is fine enough so as not to filter out waves of short wavelengths. In terms of boundary conditions in the boundary value problems, tied degrees of freedom were used along the vertical boundaries of the mesh during the dynamic analysis (Zienkiewicz et al., 1988) to ensure 1D conditions, whereas vertical displacements were restricted along the bottom boundary. The acceleration was applied incrementally along the base of the mesh. A modified Newton-Raphson scheme employing a sub-stepping stress point algorithm was the non-linear solver (Potts and Zdravković, 1999), while the generalised  $\alpha$ -method of Chung and Hulbert (1993) with a spectral radius at infinity,  $\rho_\infty$ , of 0.42 was used as the time-integration scheme. A time step of  $\Delta t=0.01$  s was found to be small enough to achieve an accurate solution.

Sand behaviour was modelled using a two-surface bounding surface plasticity model, which can realistically simulate liquefaction. The model is based on the Papadimitriou and Bouckovalas (2002) modified version of the original two-surface model proposed by Manzari and Dafalias (1997), extended so as to tackle cyclic loading and complex dynamic phenomena involving a range of cyclic strain amplitudes. The model has been implemented in ICFEP in generalized three-dimensional stress space (Taborda, 2011; Taborda et al., 2014) and includes several alterations which improve various aspects of its capabilities. These include a power law for the Critical State Line, an altered expression of the hardening modulus and the introduction of a secondary yield surface to improve the numerical stability of the model.

Table 2 presents the parameters for Nevada Sand, as established by Taborda (2011). A total of 4 resonant column tests, 2 1-D consolidation/rebound tests, 22 undrained and drained monotonic triaxial compression and extension and 5 undrained cyclic triaxial tests were used for the calibration of the model (Arulmoli et al., 1992).

Table 2. Model parameters for Nevada Sand (Taborda, 2011).

Parameter	Value	Parameter	Value	Parameter	Value
$p'_{ref}$	1000.0 kPa	$A_o$	1.46	$h_o$	1.939
$(e_{cs})_{ref}$	0.887	$m$	0.065	$\gamma$	1.214
$\lambda$	0.14	$p'_{YS}$	1.0 kPa	$e_{max}$	0.818
$\xi$	0.25	$B$	164.0	$\alpha$	1.0
$M_c^c$	1.29	$a_1$	0.3	$\beta$	0.0
$M_e^c$	0.9	$\kappa$	2.0	$\mu$	1.5
$k_c^b$	2.18	$\gamma_1$	6.5E-04	$H_o$	314.6
$k_c^d$	2.35	$\nu$	0.2	$\zeta$	1.59

### Results of site response analyses

The results of the coupled site response FE analysis in terms of initial and final mean effective stress profiles of the deposit, as well as of the stress path at 7 m depth are shown in Figure 4(a) and 4(b), respectively. The 1999 Chi-Chi acceleration time-history results in complete liquefaction of the 15 m NS deposit at the end of the 66 s of the motion duration, with de-amplification of the acceleration time-history towards the surface.

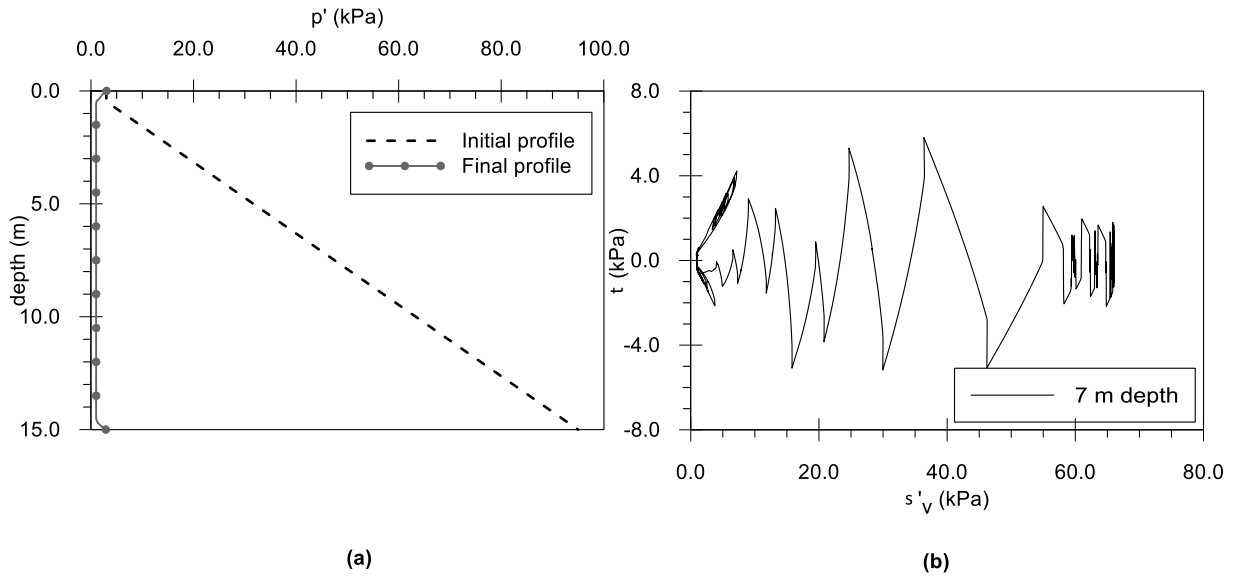


Figure 4. Results of coupled site response analysis in terms of (a) mean effective stress profile of the 15 m NS deposit and (b) stress path in vertical effective – shear stress space at 7 m depth

Figure 5 shows the surface acceleration time-history from the drained analysis, which was converted to obtain the number of equivalent uniform cycles. The shear stress time-history obtained at 7 m depth from the drained analysis is also plotted in the same Figure which has a maximum shear stress,  $\tau_{max}$ , of 9.8 kPa. This agreed well with the value obtained from the empirical relationship of Seed & Idriss (1971) according to Equation 2:

$$\tau_{max} = \frac{\alpha_{max}\sigma_{vo}}{g} \cdot r_d \quad (2)$$

where  $\alpha_{max}$  is the maximum surface acceleration,  $\sigma_{vo}$  is the total vertical stress at the point of interest,  $g$  is the gravitational acceleration and  $r_d$  is a shear stress reduction coefficient to account for the deformability of the soil column above the point of interest, given by Equations 3, 4 and 5 (Idriss, 1999):

$$r_d = e^{(\alpha(z)+\beta(z)M)} \quad (3)$$

$$\alpha(z) = -1.012 - 1.126 \sin\left(\frac{z}{11.73} + 5.133\right) \quad (4)$$

$$\beta(z) = 0.106 + 0.118 \sin\left(\frac{z}{11.28} + 5.142\right) \quad (5)$$

where  $z$  is depth in metres and  $M$  is moment magnitude.

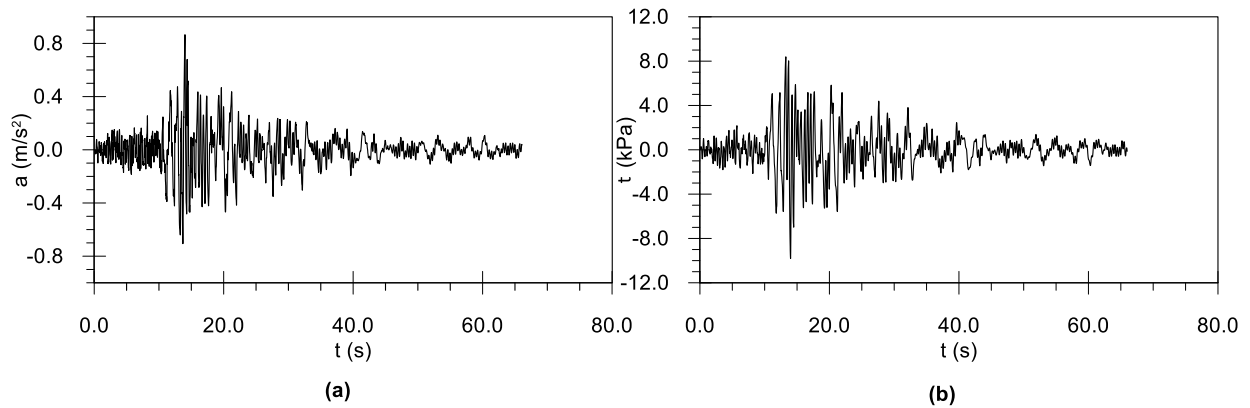


Figure 5. Results of drained analysis in terms of (a) surface acceleration time-history and (b) shear stress time-history at 7 m depth

### Results of undrained simple shear test simulations

Figures 6, 7 and 8 show the results in terms of effective stress paths and excess pore pressure ratio development for the various single element USS tests conducted in ICFEP. It should be emphasised that all the combinations of number of cycles and stress amplitudes are, according to Seed's et al. (1975) methodology, equivalent, meaning that any of these loading patterns should exhibit similar results: liquefaction of the element at 7 m depth in the deposit, in accordance with the results of the coupled FE analysis. In the following the criterion used to identify initial liquefaction is a value of the excess pore water pressure ratio,  $r_u$ , of 0.95.

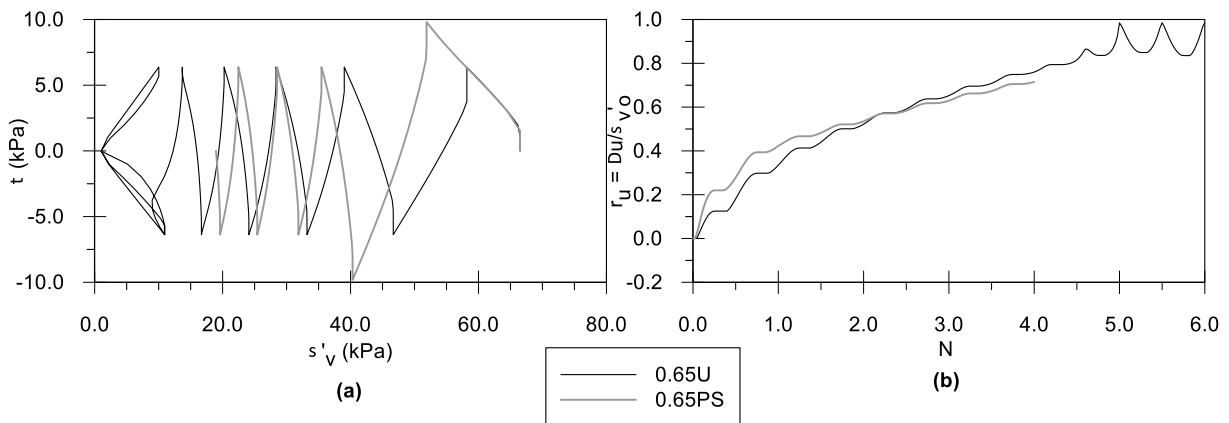


Figure 6. (a) Effective stress paths in vertical stress – shear stress space and (b) excess pore pressure ratio  $r_u$  evolution for the USS test with uniform shear stress amplitude cycles at  $0.65T_{max}$  (0.65U) and with a peak shear stress cycle followed by three cycles at  $0.65T_{max}$  (0.65PS)

Figure 6 shows that the USS test consisting of six cycles at  $0.65T_{max}$  (denoted as 0.65U) takes the stress state above the PTL after about 4.75 cycles, leading to fabric loss, a sudden increase in the excess pore water pressure ratio to values close to unity and to a very compliant stress path on unloading with a dramatic decrease in mean effective stress,  $p'$ . Complete liquefaction takes place after 5 cycles of loading. Conversely, the non-uniform USS test of four cycles with the peak stress cycle preceding the three cycles at  $0.65T_{max}$  (denoted as 0.65PS) does not lead to complete loss of strength, as none of the cycles results in crossing of the PTL and subsequent fabric loss. When the peak stress cycle is placed at the end of the loading sequence (test 0.65PE in Figure 7), however, the PTL is once again crossed, with triggering of liquefaction taking place just at the end of the fourth cycle.

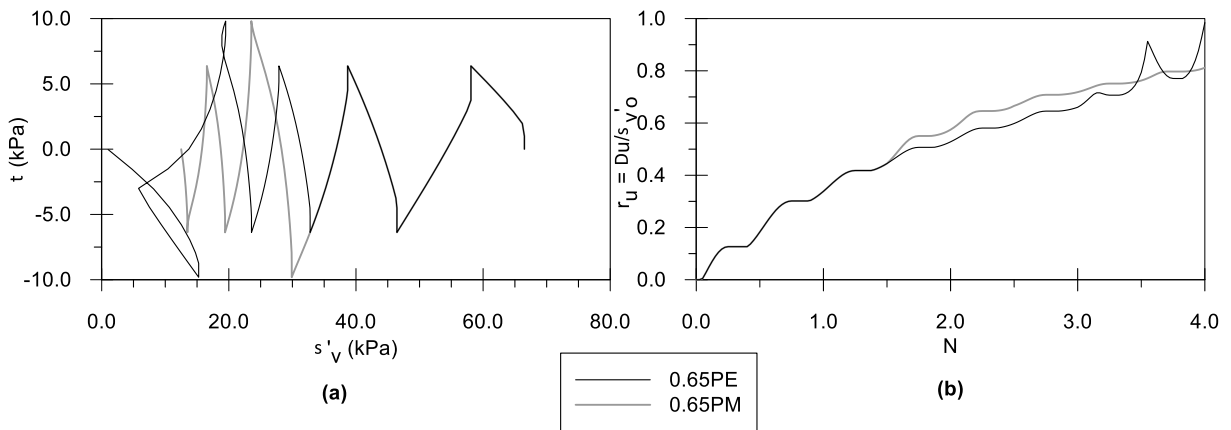


Figure 7. (a) Effective stress paths in vertical stress – shear stress space and (b) excess pore pressure ratio  $r_u$  evolution for the USS test with a peak shear stress cycle in-between the cycles of  $0.65\tau_{max}$  amplitude (0.65PM) and with the peak shear stress cycle following the three cycles at  $0.65\tau_{max}$  (0.65PE)

The results of the USS test with the peak shear stress cycle in-between the smaller cycles of  $0.65\tau_{max}$  have also been included in Figure 7 (0.65PM). Clearly, as the initial stress and loading conditions of the first 1.5 cycles are identical between tests 0.65PE and 0.65PM, the initial part of the response is indistinguishable. Nevertheless, the occurrence of the peak stress cycle in the middle of the loading history does not take the stress state above the PTL, and the remaining cycles are not sufficient to lead to liquefaction.

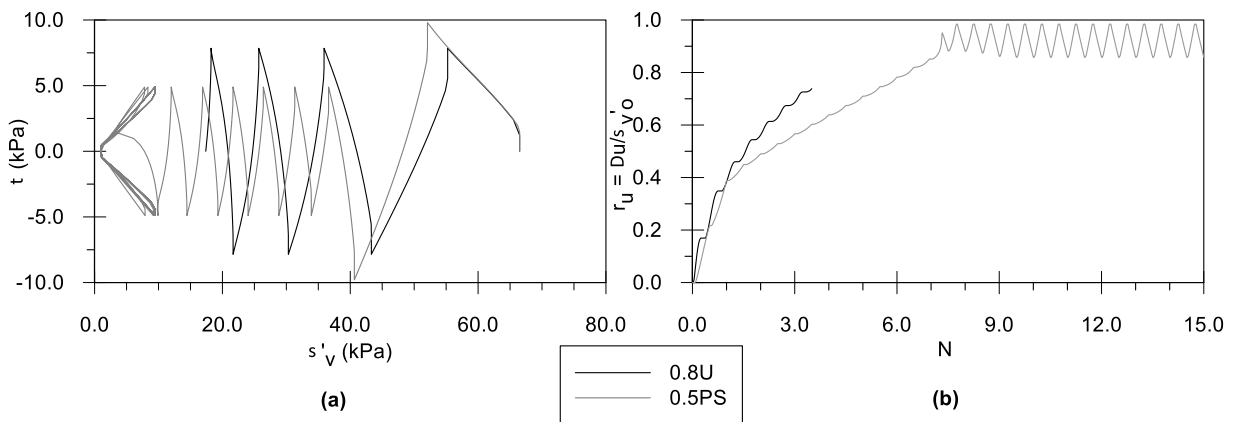


Figure 8. (a) Effective stress paths in vertical stress – shear stress space and (b) excess pore pressure ratio  $r_u$  evolution for the USS test with a peak shear stress cycle followed by 14 cycles at  $0.5\tau_{max}$  amplitude (0.5PS), as well as for the USS test with uniform stress amplitude cycles at  $0.8\tau_{max}$  (0.8U)

The results of the non-uniform test 0.5PS, with the peak shear stress cycle at the beginning of the loading followed by fourteen cycles of amplitude of  $0.5\tau_{max}$  are shown in Figure 8. Although the peak cycle is not strong enough to take the stress state above the PTL, the remaining cycles at  $0.5\tau_{max}$  result in crossing of the PTL after 7.5 cycles, marking an abrupt rise of the excess pore pressures and the onset of a compliant response after stress reversal, with the  $r_u$  taking a value of 95% after about 8 cycles. As expected, the results of the two tests with the peak shear stress cycle in-between or at the end of the  $0.5\tau_{max}$  stress amplitude cycles also result in complete loss of effective stress and are not shown here for brevity. Finally, the USS test consisting of three and a half cycles at  $0.8\tau_{max}$  (denoted as 0.8U in Figure 8) once again does not lead the stress state above the PTL and as a result liquefaction is not triggered.



The above results clearly show that, contrary to what is stated in the Seed et al. (1975) methodology, each point on the normalised CRR- $N_L$  curve is not equivalent. Therefore, one would have to carefully select the amplitude of the uniform shear stress cycles to be applied in laboratory tests to avoid over-predicting the cyclic strength of sands when subjected to irregular seismic loading in the field. Additionally, as soil is non-linear and has a stress path-dependent response, the amplitude of the peak shear stress cycle and its location within the loading time-history can affect the occurrence of plastic dilation and subsequent liquefaction, something which may not be captured properly when uniform amplitude stress cycles are employed. Previous studies in which the peak shear stress cycle, irrespective of its location, was large enough to take the stress state above the phase transformation line, with the rest of the loading sufficient to lead to initial liquefaction, have shown that, the earlier the peak is applied in the loading pattern, the more the cyclic strength of sands is decreased. In the current study, when the peak shear stress cycle is applied in the early stages or in the middle part of the loading pattern, its amplitude is not sufficient to lead to plastic dilation. Therefore, depending on the amplitude of the remaining cycles and their corresponding number, as obtained through the Seed et al. (1975) methodology, initial liquefaction may or may not take place. Conversely, when the peak shear stress cycle is applied at the end of the loading pattern, the PTL is crossed, leading to a rapid reduction of the mean effective stress on stress reversal and initial liquefaction.

### Conclusions

This numerical study focuses on the irregularity of seismic loading and on its potential implications on the laboratory evaluated liquefaction resistance of sands from uniform element testing. The results suggest that Miner's (1945) concept, on which Seed's et al. (1975) methodology for the equivalent number of cycles of constant stress amplitude is based, cannot adequately describe the behaviour of non-linear sands under strong ground motion during which large irreversible strains develop. The location of the peak stress cycle in the loading sequence is of profound importance to the response of sand to liquefaction, as this may result in the crossing of the Phase Transformation Line, leading to fabric changes and to a very compliant response after stress reversal. Single element undrained simple shear test simulations of uniform stress amplitude cycles of different percentages of the maximum shear stress registered in the shear stress time-history were also carried out in this study. These showed that different points on the normalised curve of cyclic resistance ratio (CRR) versus number of cycles to liquefaction ( $N_L$ ) from the Seed et al. (1975) methodology are not necessarily equivalent. This implies that careful consideration of the stress level chosen in uniform cyclic laboratory testing is required, in order to avoid over-prediction of the liquefaction resistance of sands under real seismic conditions. In the future, it would also be interesting to extend this study to a range of ground motions of different intensities.

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